Mathematics

MATHCC – I/ Calculus and Geometry

1st Semester / FM – 60

Group – A

[Answer any four questions given bellow **(4 X 3 = 12)**]

1. Find the pedal equation of the parabola $\frac{2a}{r} = 1 - \cos\theta$.
2. Show that the radius of curvature of the at any point of the cardioid $r = a(1 - \cos\theta)$ varies as $\sqrt{r}$.
3. Find the envelope of the family of lines $y = mx + \sqrt{1 + m^2}$, m being a parameter.
4. If $I_n = \int_0^{\pi/2} x^n \sin x\, dx$ and $n > 1$, show that $I_n + n(n-1)I_{n-2} = n\left(\frac{\pi}{2}\right)^{n-1}$.
5. Show that the tangents at the extremities of a focal chord of a parabola meet at right angle on the directrix.
6. A plane passes through a fixed point (p, q, r) and cuts the axes in A, B, C. Show that the locus of the centre of the sphere $OABC$ is $\frac{p}{x} + \frac{q}{y} + \frac{r}{z} = 2$.

Group – B

[Answer any four questions given bellow **(4 X 6 = 24)**]

7. If $y = (\sin^{-1} x)^2$, show that $(1 - x^2)y_{n+2} - (2n+1)xy_{n+1} - n^2 y_n = 0$. Hence find $(y_n)_0$.
8. Evaluate the limit: $\lim\limits_{x\to 0}\left(\frac{\sin x}{x}\right)^{1/x^2}$.
9. Show that the radius of curvature of the curve given by $x^2 y = a\left(x^2 + \frac{a^2}{\sqrt{5}}\right)$ is least for the point $x = a$ and its value there is $\frac{9}{10}a$.
10. Show that the area common to the cardioid $r = a(1 + \cos\theta)$ and the circle $r = \frac{3}{2}a$ is $\left(\frac{7}{4}\pi - \frac{9\sqrt{3}}{8}\right)a^2$.
11. Discuss the nature of the conic represented by $3x^2 - 8xy - 3y^2 + 10x - 13y + 8 = 0$. Also find its canonical form.
12. Show that the locus of the points from which three mutually perpendicular lines can be drawn to intersect the conic $z = 0, ax^2 + by^2 = 1$ is $ax^2 + by^2 + (a+b)z^2 = 1$.

Group – C

[Answer any two questions given bellow **(2 X 12 = 24)**]

13. (i). If $V_n = \frac{d^n}{dx^n}(x^n \log x)$, show that $V_n = nV_{n-1} + (n-1)!$
    Hence show that $V_n = n!\left(\log x + 1 + \frac{1}{2} + \frac{1}{3} + \cdots + \frac{1}{n}\right)$.
    (ii). Find the asymptotes of the curve $x^3 + 3x^2y - 4y^3 - x + y + 3 = 0$. (6+6)
14. (i). Find the values of a and b such that $\lim_{x \to 0} \frac{x(1+a\cos x) - b\sin x}{x^3} = 1$.
    (ii). Find a reduction formula for $I_{m,n} = \int_0^{\pi/2} \cos^m x \sin nx \, dx$, m, n being positive integers.
    Hence deduce that $I_{m,n} = \frac{1}{2^{m+1}}\left[2 + \frac{2^2}{2} + \frac{2^3}{3} + \cdots + \frac{2^m}{m}\right]$. (6+6)

15. (i). Prove that the volume of the solid obtained by revolving the lemniscate $r^2 = a^2 \cos 2\theta$ about the initial line is $\frac{1}{2}\pi a^3 \left\{\frac{1}{\sqrt{2}} \log(\sqrt{2}+1) - \frac{1}{3}\right\}$.
    (ii). If PSP' and QSQ' are two perpendicular focal chords of a conic, prove that
    $\frac{1}{PS.SP'} + \frac{1}{QS.SQ'} =$ constant. (6+6)

16. (i). Prove that the locus of points of intersection of pair of perpendicular tangents to the circle $x^2 + y^2 = a^2$ is $x^2 + y^2 = 2a^2$.
    (ii). Show that the generators of the hyperboloid $\frac{x^2}{25} + \frac{y^2}{16} - \frac{z^2}{4} = 1$ which are parallel to the plane $4x - 5y - 10z + 7 = 0$ are $x + 5 = 0, y + 2z = 0$ and $y + 4 = 0, 2x = 5z$. (6+6)

Mathematics

MATHCC-II/ALGERBRA
2nd Semester/FM-60

GROUP- A

Answer any four questions from the following (4X3=12)

1. Use Euclidean algorithm to find the integers x and y such that $gcd(72, 120) = 72x + 120y$.

2. If the function $f : R \to R$ be defined by $f(x) = x^2 + 1$, for $x \in R$, then find $f^{-1}(-1)$ and $f^{-1}(3)$.

3. Find the inverse of the matrix using Caylay-Hamilton theorem $\begin{pmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{pmatrix}$.

4. Find the rank of the matrix by reducing it to its normal form $\begin{pmatrix} 1 & 3 & 6 & -1 \\ 1 & 4 & 5 & 1 \\ 1 & 5 & 4 & 3 \end{pmatrix}$.

5. Let $f : A \to B$ be a mapping. A relation R is defined on A by xRY if and only if $f(x) = f(y)$, $x, y \in A$". Show that R is an equivalence relation. 3+4

6. If X and Y are two non-empty sets and $f : X \to Y$ is a mapping, then for any subsets A and B of X , prove that $f(A \cup B) = f(A) \cup f(B)$ and $f(A \cap B) \subseteq f(A) \cap f(B)$.

GROUP- B

Answer any four questions from the following (4X6=24)

7. Solve the equation $3x^4 + x^3 + 4x^2 + x + 3 = 0$, which has four distinct roots of equal moduli.

8. If α, β, γ be the roots of the equation $x^3 + px^2 + qx + r = 0$, find the value of $\sum \alpha^2\beta^2$.

9. Solve the equation $x^3 - 5x^2 - 16x = -80$, the sum of two of its root is zero. Also solve the equation $x^7 - x = 0$.

10. Find all the eigen values and eigen vectors of the matrix $\begin{pmatrix} 1 & -1 & 4 \\ 3 & 2 & -1 \\ 2 & 1 & -1 \end{pmatrix}$.

11. Find for what values of α and β the following system of equations has (i) a unique solution, (ii) no solution, (iii) infinite number of solutions over the field of rational numbers

$$x + 4y + z = 1, 2x + 7y + 5z = 2\beta, 4x + \alpha y + 10z = 2\beta + 1$$

12. Let z be a variable complex number such that $\mid z - 10/z \mid = 3$. Find the greatest and least value of $\mid z \mid$.

GROUP- C

Answer any two questions from the following (2X12=24)

13. (i) Find the row space and column space of the following matrix $\begin{pmatrix} 0 & -1 & 0 \\ 3 & 0 & -1 \\ 7 & 1 & 1 \end{pmatrix}$.
(ii) Solve the equation $x^4 - 4x^2 + 8x + 35 = 0$, if one root is $2 + i\sqrt{3}$.

14. (i) If the roots of the equation $x^3 + px^2 + qx + r = 0$ are in A.P, then show that $2p^3 - 9pq + 27r = 0$.
(ii) Solve the equation $27x^3 + 42x^2 - 28x - 8 = 0$ if roots are in G.P.

15. (i) Prove that injective mapping from a finite set to itself is a bijective mapping.
(ii) If $f : A \to B$, $g : B \to C$ and $h : B \to C$, then prove that $h(gf) = (hg)f$.

16. A mapping $f : A \to B$ is
(i) injective if and only if f has left inverse
(ii) surjective if and only if f has right inverse.

Mathematics
MATHCC – III/ Real Analysis
2nd Semester / FM – 60

Group – A

[Answer any four questions given bellow **(4 X 3 = 12)**]

1. Prove or disprove: the set $\mathbb{Q}$ of rational numbers is complete.
2. Let A and B be two bounded sets in $\mathbb{R}$ and $X = \{a - b : a \in A, b \in B\}$. Prove that sup. X = sup. $A -$ sup. B .
3. Find the closure of the set $\bigcap_{n=1}^{\infty} \left(-\frac{1}{n}, 1 + \frac{1}{n}\right)$.
4. Find the limit points of the sequence $\left\{\left(1 + \frac{1}{n}\right) 2^{(-1)^n}\right\}$.
5. Use Cauchy criterion to prove that the sequence $\left\{1 + \frac{1}{2} + \frac{1}{3} + \cdots + \frac{1}{n}\right\}$ is not convergent.
6. Show that a necessary condition for convergence of an infinite series $\sum u_n$ is that $\lim_{n\to\infty} u_n = 0$. Hence show that the series $\frac{1}{2} + \frac{2}{3} + \frac{3}{4} + \cdots$ is not convergent.

Group – B

[Answer any four questions given bellow **(4 X 6 = 24)**]

7. Define a countable set. Prove that a countable union of countable sets is countable.
8. Give the definition of limit point. Let ξ be a limit point of a set S. Prove that every neighbourhood of ξ contains infinitely many points of S.
9. Define a compact set. Prove that every closed and bounded subset of $\mathbb{R}$ is compact.
10. Let $\{a_n\}, \{b_n\}, \{c_n\}$ be three sequences such that $a_n \le b_n \le c_n$ and $\lim a_n = \lim c_n = l$. Show that $\lim b_n = l$.
11. Show that the sequence $\{S_n\}$, defined by the recursion formula $S_{n+1} = \sqrt{3S_n}$, $S_1 = 1$, converges to 3.
12. Prove that the positive term series $\sum \frac{1}{n^p}$ converges if $p > 1$ and diverges if $p \le 1$.

Group – C

[Answer any two questions given bellow **(2 X 12 = 24)**]

13. (i). Prove that every bounded infinite subset of $\mathbb{R}$ has a limit point.

    (ii). Define closure of a set. Prove that the closure a set S is the smallest closed set containing S. (6+(1+5))

14. (i). Prove that a set S is closed iff its complement is open.

    (ii). State and prove Cauchy's first theorem on limit of a sequence. Use this theorem to prove that $\lim_{n\to\infty} \frac{1}{n}\left[1 + 2^{1/2} + 3^{1/3} + \cdots + n^{1/n}\right] = 1.$ (4+(6+2))

15. (i). Let $\{S_n\}$ be a monotone increasing sequence which is bounded above. Prove that $\{S_n\}$ converges to its least upper bound.

    (ii). Test the convergence of the hypergeometric series

$$1 + \frac{\alpha\beta}{1.\gamma}x + \frac{\alpha(\alpha+1)\beta(\beta+1)}{1.2\,\gamma(\gamma+1)}x^2 + \frac{\alpha(\alpha+1)(\alpha+2)\beta(\beta+1)(\beta+2)}{1.2.3\,\gamma(\gamma+1)(\gamma+2)}x^3 + \cdots$$

    For all positive values of x; α, β, γ being all positive. (6+6)

16. (i). State and prove Cauchy's root test.

    (ii). Test the convergence of the following series:

    (a) $\sum \frac{1}{n^{1+1/n}}$

    (b) $\sum \left(1 + \frac{1}{\sqrt{n}}\right)^{-n^{3/2}}$ (6+(3+3))

Mathematics

MATHCC – IV/ Differential Equations and Vector Calculus

2nd Semester / FM – 60

Group – A

[Answer any four questions given bellow **(4 X 3 = 12)**]

1. Prove that the differential equation of all circles touching the y-axis at the origin is
$$(y^2 - x^2)dx - 2xy\,dy = 0.$$
2. Show that the functions $e^x, \sin x, \cos x$ are linearly independent.
3. Solve the equation: $x\frac{dy}{dx} + y = y^2 \log x$.
4. If $\vec{r} = a\cos t\,\hat{\imath} + a\sin t\,\hat{\jmath} + bt\,\hat{k}$, show that $\left|\frac{d\vec{r}}{dt} \times \frac{d^2\vec{r}}{dt^2}\right|^2 = a^2(a^2 + b^2)$.
5. A particle moves along the curve $x = 2t^2, y = t^2 - 4t, z = 3t - 5$. Find the components of velocity and acceleration at time $t = 1$ in the direction of $\hat{\imath} - 3\hat{\jmath} + 2\hat{k}$.
6. If $\vec{F} = (3x^2 + 6y)\hat{\imath} - 14yz\,\hat{\jmath} + 20zx^2\hat{k}$, then evaluate $\int_C \vec{F}.d\vec{r}$, from (0,0,0) to (1,1,1) along the curve $x = t, y = t^2, z = t^3$.

Group – B

[Answer any four questions given bellow **(4 X 6 = 24)**]

7. In a certain culture of bacteria, the rate of increase is proportional to the number present. If it be found that their number doubles in 4 hours, then use mathematical models to find their number at the end of 12 hours.
8. Solve the equation: $\frac{dy}{dx} - \frac{1}{1+x}\tan y = (1+x)e^x \sec y$.
9. Solve the equation: $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = x^2e^{3x}$.
10. Solve the simultaneous linear differential equations:
$$t\,dx = (t - 2x)dt$$
$$t\,dy = (tx + ty + 2x - t)dt$$
11. Find the directional derivative of $\varphi = 2xy - z^2$ at $(2, -1, 1)$ in the direction of $3\hat{\imath} + \hat{\jmath} - \hat{k}$. In what direction is the directional derivative maximum? What is the value of the maximum?
(2+2+2)
12. Evaluate $\int_C \vec{F}.d\vec{r}$, where $\vec{F} = (x^2 - 3y^2)\hat{\imath} + (y^2 - 2x^2)\hat{\jmath}$ and C is the closed curve in the xy −plane, given by $x = 3\cos t, y = 2\sin t, 0 \le t \le 2\pi$ and C is described in the anti-clock-wise sense.

Group – C

[Answer any two questions given bellow **(2 X 12 = 24)**]

13. (i). Transform the differential equation $\frac{dy}{dx}+\frac{y}{x}\log y=\frac{y}{x^2}(\log y)^2$ into a linear form and solve it.

(ii). Solve the equation: $x^2\frac{d^2y}{dx^2}-3x\frac{dy}{dx}+4y=2x^2$ (6+6)

14. (i). Show that a differential equation of the form $[y+x\,f(x^2+y^2)]dx+[y\,f(x^2+y^2)-x]dy=0$ is not exact. Show that $\frac{1}{x^2+y^2}$ is an integrating factor of an equation of this form. Hence solve $[y+x(x^2+y^2)^2]dx+[y\,(x^2+y^2)^2-x]dy=0$.

(ii). Find the general solution of the following equation using method of undetermined coefficients: $\frac{d^2y}{dx^2}-3\frac{dy}{dx}=x+e^x\sin x$. ((2+2+2)+6)

15. (i). Apply the method of variation of parameters to solve the following equation:

$$\frac{d^2y}{dx^2}-3\frac{dy}{dx}+2y=\frac{e^x}{1+e^x}$$

(ii). If $\vec{a}$ be a constant vector, then show that

(a). $\vec{\nabla}\times\left(\vec{a}\times\vec{b}\right)=\vec{a}\,\mathrm{div}\,\vec{b}-\vec{a}.\vec{\nabla}\vec{b}$

(b). $\vec{\nabla}\left(\vec{a}.\vec{b}\right)=\vec{a}.\vec{\nabla}\vec{b}+\vec{a}\times\mathrm{curl}\,\vec{b}$ (6+(3+3))

16. (i). Prove the following identities:

(a). $\left[\vec{a}+\vec{b},\ \vec{b}+\vec{c},\ \vec{c}+\vec{a}\right]=2\left[\vec{a}\vec{b}\vec{c}\right]$

(b). $\left[\vec{a}\times\vec{b},\ \vec{b}\times\vec{c},\ \vec{c}\times\vec{a}\right]=\left[\vec{a}\vec{b}\vec{c}\right]^2$

(ii). Prove that a necessary and sufficient condition that the vector field defined by the vector point function $\vec{F}$ with continuous derivatives be conservative is that $\vec{\nabla}\times\vec{F}=\vec{0}$.

((3+3)+6)

MATHEMATICS

MATHCC-V (THEORY OF REAL FUNCTIONS AND INTRODUCTION OF THE METRIC SPACE)

3RD SEMESTER / FM-60

GROUP A

Answer any four questions from the following (4×3=12)

1. Prove that $\lim_{x\to\infty}\frac{1}{x}=0$
2. Let k ∈ R .Prove that the function f defined by f(x)= k, x∈ R is continuous on R.
3. Every differentiable function is continuous –justify this statement.
4. Prove that the limit of f(x)=sin1/x does not exist when x→ 0.
5. Give an example of a function f which satisfies the intermediate value property on a closed and bounded interval [a, b] but is not continuous on [a, b].
6. Let (X,d) be a metric space for any x,y ∈ X define $d_1(x,y)=\min\{1, d(x,y)\}$.show that d_1 is a metric on X.

Group B

Answer any four questions from the following (4×6=24)

1. Using Cauchy principle prove that $\lim_{x\to 0} cos1/x$ does not exist
2. Let I=(a,b) be a bounded open interval and f:I→ R be a monotone increasing function on I then prove the following
   i. If f be bounded above on I then $\lim_{x\to b-} f(x)$ =sup f(x), x∈ (a,b).
   ii. If f be bounded below on I the $\lim_{x\to a+} f(x)$=inf f(x), x∈ (a,b).
3. A function is defined on [0,1] by f(0)=1 and f(x)= 0 when x is rational and
   =1/n, when x=m/n , and gcd(m,n)=1
   Prove that f is continuous at every irrational point in [0,1] and discontinuous at every rational point in [0,1].
4. State and prove Rolle's theorem.
5. Prove that $\frac{x}{1+x} < \log(1+x) < x$ for all x > 0.
6. In a metric space .show that finite intersection of open set is open set.

Group C

Answer any four questions from the following (2×12=24)

1. a. Let D⊂ R and f, g, h be functions on D to R. Let $c\epsilon D'$.prove that if f(x)≤ $g(x) \le h(x)$ for all x∈ $D-\{c\}$ and if $\lim_{x\to c} f(x)=l$ then $\lim_{x\to c} g(x)=l$.

b. Show that $\lim_{x \to 0} x cos 1/x = 0$

2. a. Let D$\subset R$ and f:D$\to R$ be a function. Let c$\in D \cap D'$. Prove that f is continuous at c iff for every sequence $\{x_n\}$ in D converging to c , the sequence $\{f(x_n)\}$ converges to f(c).

b. Let [a,b] be a closed and bounded interval and a function f:[a,b]$\to R$ be continuous on [a,b]. Prove that f(a)$\neq f(b)$then f attains every value between f(a) and f(b) at least once in the interval (a,b).

3. a. State and prove Mean value theorem for a function f: [a,b]$\to R$.

b. Prove that between any two real roots of the equation e^xcosx+1=0 there is at least one real root of the equation e^xsinx+1=0.

4. a. Show that A closed subset of a complete metric space is a complete subspace.

b. Prove that a subset A of a metric space M is closed iff it contains all its limit points.

MATHEMATICS

MATH-CC-VI (GROUP THEORY I)

3rd SEMESTER / F.M -60

GROUP A

ANSWER ANY FOUR QUESTIONS FROM THE FOLLOWING ($4\times 3 = 12$)

1. Express the permutation $\begin{pmatrix} 1\,2\,3\,4\,5\,6\,7\,8 \\ 4\,5\,1\,3\,2\,8\,6\,7 \end{pmatrix}$ as the product of disjoint cycles .what will be the order of this permutation .Determine whether the permutation is even or odd.
2. Prove that in a group (G,*) each element has unique inverse .
3. Describe Klein's 4 group.
4. Find the centre of the group Q_8.
5. Prove that (Q,+) is a non cyclic group.
6. Show that SL(2,R) is a subgroup of GL(2,R).

GROUP B

ANSWER ANY FOUR QUESTIONS FROM THE FOLLOWING ($4\times 6 = 24$)

1. Prove that a non empty subset H is subgroup of a group G if and only if a,b$\in H$

   $ab^{-1}\in H$.

2. Let (G, *) be a cyclic group generated by a .then prove that G is infinite if and only if order of a is infinite.
3. Prove that every group of prime order is cyclic.
4. Show that centre of a group is a normal subgroup of G.
5. State and prove the First isomorphism theorem .

GROUP C

ANSWER ANY TWO QUESTIONS FROM THE FOLLOWING ($2\times 12 = 24$)

1. a. Let H,K be subgroups of a group G. prove that HK is a subgroup of G iff HK=KH

   b. Show that centralizer of an element of a group is a group .

2. a. Prove that every subgroup of a cyclic group G is cyclic .

   b. prove that a cyclic group of finite order n has a subgroup of order d for every positive divisor d of n.

3. a. State and prove the Lagrange's theorem for a group G .

   b. If p be prime and a be a positive integer such that p is not a divisor of a the prove that

   $a^{p-1}\cong 1(mod p)$

4. a. Show that intersection of two normal subgroup of a group G is is normal in G

   b. Show that there does not exist an onto homomorphism from klein's 4 group to to (Z_4,+).

Mathematics

MATHCC-VII/Riemann integration and series of functions
3rd Semester/FM-60

GROUP- A

Answer any four questions from the following (4X3=12)

1. If f is a positive function on $(0, \infty)$ such that
(a) $f(x+1) = xf(x)$
(b) $f(1) = 1$
(c) $Logf$ is convex function
then show that $f(x) = \Gamma(x)$.

2. Show that the sequence $\{f_n(x) = \frac{x}{1+nx}\}$ is uniformly converges to a function f and that the equation
$f^{'}(x) = lim_{n\to\infty} f_n^{'}(x)$
is correct if $x \neq 0$ and false if $x = 0$.

3. (i) Let $4f_n(x) = tan^{-1}nx, x \in [0,1]$. Prove that the sequence f_n is not uniformly convergent on $[0,1]$.
(ii) State fundamental theorem of integral calculus.

4. Let $I = [a,b]$ and let $f : I \to R$ be a bounded function. Then the lower integral $L(f)$ and the upper integral $U(f)$ of f on I exist. Moreover, $L(f) \leq U(f)$.

5. Define radius of convergence and internal of convergence. Find the radius of convergence of the following
$\sum 2^{2n}x^{2n}$.

6. Define beta and gamma function. Find the radius of convergence of the series $\sum a_n x^{2n}$ if radius of convergence of the series $\sum a_n x^n$ is R.

GROUP- B

Answer any four questions from the following (4X6=24)

7. Prove the following
(i) The functional equation $\Gamma(x+1) = x\Gamma(x)$ holds for $0 < x < \infty$.
(ii) $\Gamma(n+1) = n!, n = 1, 2, 3, 4, ...$
(iii) $Log\Gamma$ is convex on $(0, \infty)$.

8. Let (f_n) be a sequence of functions in $R[a, b]$ and suppose that (f_n) converges uniformly on $[a, b]$ to f. Then prove that $f \in R[a, b]$.

9. If the function f on the interval $I = [a, b]$ is either continuous or monotone on I, then f is Darboux integrable on I.

10. Let f be real values function define on $[a, b]$. the prove the following
(i) if f is Riemann integrable then $\mid f \mid$ is Reimann integrable. what about the converse. Justify your answer.
(ii) if f is Riemann integrable then f^2 is Reimann integrable.

11. State and prove necessary and sufficient condition of Riemann integrability.

12. GROUP- C

Answer any two questions from the following (2X12=24)

13. Obtain the Fourier series of the following function defined in $(0, 2\pi)$

$$f(x) = \begin{cases} x, & \text{if } 0 < x < \pi \\ \pi, & \pi < x < 2\pi. \end{cases} \text{ and has period } 2\pi.$$

14. (i) State Cauchy Hadamard theorem and weierstrass approximation theroem.
(ii) Define piecewise continuous function. Is piecewise continuous function is Riemann integrable? Justify your answer.

15. (i) For each of the following series, discuss whether the series is convergent or divergent
(a) $\sum (n!)^{\frac{1}{n}}$, (b) $\sum \frac{1}{n} sin\frac{n\pi}{2}$
(ii) For each of the following series, determine the values of $x \in R$ for which the series is convergent
(a) $\sum sin(n\pi x)$, (b) $\sum cos\frac{nx}{n^2}$

16. (i) Test the convergence of $\int_0^1 x^m(1-x)^{1-n}dx$

(ii) Show that $\int_2^\infty \frac{cox}{logx}$ is conditionally convergent

MATHEMATICS

MATHSEC1-GRAPH THEORY

3RD SEMESTER / FM-60

GROUP- A

ANSWER ANY FOUR QUESTIONS FROM THE FOLLOWING(4×3=12)

1. What is a graph ? give two example of graph.
2. In any graph G=(V,E, φ), Prove that $\Sigma d(V) = 2|E|$.
3. Examine whether a simple graph having degree sequence (2,2,4,5,5) exists.
4. Prove that size of every connected graph of order n is at least n-1.
5. Find n , for which the complete graph K_n is semi –Eulerian and Eulerian .
6. Give an example of a graph that is Eulerian and Hamiltonian .

Group B

ANSWER ANY FOUR QUESTIONS FROM THE FOLLOWING(4×6=24)

1. Prove that a graph G contains a circuit , if degree of each vertex of G is at least two.
2. Prove that a complete graph K_n is Eulerian iff n is odd.
3. Let G be simple graph of order n having k components .Prove that the size of G can be at most ½(n-k)(n-k+1).
4. Prove that every simple graph with n($\geq$ 2)vertices must have at least one pair of vertices whose degrees are same .
5. Examine whether a regular graph with 8 vertices and seven edges exists.
6. Find the number of edges in K_n and $K_{n,m}$.

Group C

ANSWER ANY TWO QUESTIONS FROM THE FOLLOWING(2×12=24)

1a. Prove that A connected graph G is semi eulerian if and only if it has exactly two odd degree vertices

b. Let G be a simple connected graph of order n≥ 3 and sizem.Then prove that G is Hamiltonian if m$\geq 1/2(n-1)(n-2)$+2.

2 a. Find the adjacency matrix of the complete bipartite graph $K_{3,3}$

b. Draw the graph having adjacency matrix

$$\begin{matrix} 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 1 & 1 \end{matrix}$$

3. a. Prove that every circuit contains a cycle in a graph

b. In a group of seven people of Kolkata . is it possible for each person to shake hands with exactly three other people ? justify

4. a. Prove that every connected graph G remains connected after deleting an edge e from g if and only if e is a cycle edge in G.

   b. Let G be a simple graph then prove that G and its complement G' can not be both disconnected .

Mathematics

MATHCC-VIII/MULTIVARIATE CALCULUS
4th Semester/FM-60

GROUP- A

Answer any four questions from the following (4X3=12)

1. Investigate the continuity of the following function at the point
$f(x,y) = \begin{cases} x^2 + 2y, & \text{if (x,y)} \neq (1,2) \\ 0, & \text{(x,y)=(1,2).} \end{cases}$.

2. Let $f(x,y) = \begin{cases} x^2y/x^4 + y^2, & \text{if (x,y)} \neq (0,0) \\ 0, & \text{(x,y)=(0,0).} \end{cases}$. Show that $\lim_{(x,y)\to(0,0)} f(x,y)$ does not exist.

3. Evaluate the integral $\int_C (x^2dx + xydy)$ taken along the line segment from $(1,0)$ to $(0,1)$.

4. Show that $\lim_{(x,y)\to(0,0)} xy\frac{x^2-y^2}{x^2+y^2} = 0$.

5. Evaluate $\iint\limits_R (x+y)\, dx\, dy$ on the rectangle $R = [a, b; c, d]$.

6. Find the divergence $div\vec{F}$, where $\vec{F} = xy\vec{i} + yz\vec{j} + yz\vec{k}$.

GROUP- B

Answer any four questions from the following (4X6=24)

7. If $U = \phi(x+at) + \xi(x-at)$)show that $\frac{\delta^2 U}{\delta y^2} = a^2\frac{\delta^2 U}{\delta x^2}$. Also prove that if $y = x + at$ and $z = x - at$ then the equation becomes $\frac{\delta^2 U}{\delta y \delta z} = 0$.

8. Show that the function $f(x,y) = \begin{cases} xy/\sqrt{x^2+y^2}, & \text{if (x,y)} \neq 0 \\ 0, & \text{(x,y)=(0,0).} \end{cases}$ is continuous, possesses partial derivative but not differential at the origin.

9. Prove that $\int_0^1 dx \int_0^1 \frac{x-y}{(x+y)^3} dy = \frac{1}{2}$ and $\int_0^1 dy \int_0^1 \frac{x-y}{(x+y)^3} dxdy = \frac{1}{2}$.

10. Does the double integration $\iint_C \frac{x-y}{(x+y)^3} dxdy$ exist over $C = [0,1;0,1]$.

11. compute the integral $\iiint_E xyzdxdydz$ over a domain bounded by $x = 0, y = 0, z = 0, x + y + z = 1$.

12. Define the terms vector field, divergence and curl. Prove that the work done by a conservative force depends only on the begining and ending positions of the object.

GROUP- C

Answer any two questions from the following (2X12=24)

13. (i) Define directional derivative. Find the gradient of the function $f(x,y) = xy + y^2$ and use it to find the directional derivative in the direction of the vector $\vec{V} = \frac{\sqrt{3}}{2}\hat{i} + \frac{1}{2}\hat{j}$.
(ii) A rectangular box without a lid is to be made from $27m^2$ of cardboard. Find the maximum volume of such a box using the method of lagrange's multiplier.

14. (i) If $f(x,y) = \begin{cases} xytan\frac{y}{x}, & \text{if (x,y)} \neq 0 \\ 0, & \text{(x,y)=(0,0).} \end{cases}$ then show that $xf_x + yf_y = 2f$. Also find the $f_x(0,0)$ and $f_y(0,0)$ for the function $f(x,y) = \sqrt{\mid xy \mid}$.
(ii) Change the order of the following integration
$\int_0^1 dy \int_x^{\sqrt{x}} f(x,y)$.

15. (i) Prove that $\iint_R \sqrt{\mid y - x^2 \mid} dxdy = \frac{3\pi+8}{6}$, where $R = [-1,1;0,2]$.
(ii) Show that $\iint_R \sqrt{xy} dxdy = \frac{\pi}{6}$, where R is the region bounded by the lines $x = 0, y = 0, x + y = 1$.

16. (i) Compute the surface area of the sphere, $x^2 + y^2 + z^= a^2$.
(ii) Show that the repeated limit of f exist at the origin and are equal but the simulteneous limit does not exist, where $f(x,y) = \begin{cases} 1, & \text{if xy} \neq 0 \\ 0, & \text{xy=0.} \end{cases}$.

Mathematics

MATHCC-IX/RING THEORY AND LA-I

4TH Semester/FM-60

GROUP- A

Answer any four questions from the following (4X3=12)

1. Define quotient ring. Describe the quotient ring $\frac{2Z}{6Z}$

2. If R is a commutative ring with characteristic p, where p is prime, then show that $(a + b)^p = a^p + b^p$.

3. Let I be an ideal of a ring R with unity. If one of units of R belongs to I, prove that $I = R$. Give an example of an infinite ring having finite characteristic.

4. Define ideal in a ring. For $a \in R$, let $Ra = \{xa \mid x \in R\}$. Prove that Ra is a left ideal of R.

5. Let R be a ring such that $a^2 = a$ for all $a \in R$. Prove that R is a commutative ring of characteristic 2.

6. Let λ be a left ideal in a ring R and define $L(\lambda) = \{x \in R \mid xa = 0, \forall a \in \lambda\}$. Prove that $L(\lambda)$ is an ideal of R.

GROUP- B

Answer any four questions from the following (4X6=24)

7. Define ring homomorphism with example. Let $f : R \to S$ be a homomorphism of rings. Then prove that f is injective if and only if $kerf = \{0\}$.

8. Define linearly dependent and linearly independent set of vectors. Prove or disprove: Image of linearly independent set of vectors is linearly independent under any linear transformation.

9. Let V be the set of all pairs (x, y) of real numbers and let F be the field of real numbers. Define $(x, y) + (z, w) = (x + z, y + w)$ and $c(x, y) = (cx, y)$. Is V , with these operations, a vector space over the field of real numbers?

10. Define vector space with example. Prove or disprove: solution set of homogeneous differential equation is a vector space over the field of complex numbers.

11. Define basis and dimension of vector space. If V is a finite-dimensional vector space, then any two bases of V have the same (finite) number of elements.

12. Show that the vectors $\alpha_1 = (1, 0, -1)$, $\alpha_2 = (1, 2, 1)$, $\alpha_3 = (0, -3, 2)$ form a basis for R^3. Express each of the standard basis vectors as linear combinations of α_1, α_2, and α_3.

GROUP- C

Answer any two questions from the following (2X12=24)

13. Let V be the vector space consisting of all functions of the form $\alpha e^{2x}cosx$ + $\beta e^{2x}sinx$ Consider the following linear transformation $T : V \rightarrow V$ defined by $T(f) = f' + f$.

(i) Find the matrix representing of T with respect to the basis $\{e^{2x}cosx, e^{2x}sinx\}$

(ii) Use your answer from part (*i*) to find one solution to the following differential equation

$$y' + y = e^{2x}cosx$$

.

14. Prove or disprove the following

(i) An integral domain is a field.

(ii) A finite integral domain is a field.

(iii) A field is an integral domain.

(iv) A division ring is an integral domain.

(v) A division ring is a field

(vi) Every subring is ideal.

15. (i) Show that the center of a division ring is a field.

(ii) state and prove first isomorphism theorem for ring.

16. Let V and W be finite-dimensional vector spaces over the field F such that $dimV = dimW$. If T is a linear transformation from V into W, then prove that the following are equivalent

(i) T is invertible.

(ii) T is non-singular.

(iii) T is onto, that is, the range of T is W.

MATHEMATICS

MATHCC-X (METRIC SPACE AND COMPLEX ANALYSIS)

SEMESTER -4TH / F.M -60

GROUP –A

ANSWER ANY FOUR QUESTIONS FROM THE FOLLOWING (4×3=12)

1. Let $f(z)=\begin{cases}\frac{|z|}{Re(z)} if\ Re(z) \neq 0 \\ o\ if\ R(z) = 0\end{cases}$ Show that f is not continuous at z=0.
2. Show that the function f(x,y)=$\sqrt{|xy|}$ (x,y $\in R$) is not analytic at origin.
3. If f is analytic in a domain D, then show that f(z) must be constant if arg(f) is constant.
4. A function f : X → Y is continuous at a ∈ X if and only if for every neighborhood V ⊂ Y of f(a) the inverse image f^{-1} (V) ⊂ X is a neighborhood of a.
5. A metric space X is compact if and only if every collection F of closed sets in X with the finite intersection property has a nonempty intersection.
6. Show that The direct image of a compact metric space by a continuous function is compact.

GROUP B

ANSWER ANY FOUR QUESTIONS FROM THE FOLLOWING (4×6=24)

1. Show that the function f(z) =$|z|^2$ is differentiable only at z=0.
2. If f is analytic function on a region D such that Im(f)=0, then show that f is constant .
3. Let $f(z)=\begin{cases} e^{-z^2} if\ z \neq 0 \\ 0 if\ z = 0\end{cases}$ is not differentiable at 0 although f satisfies the cauchy reiman equations at 0.
4. . A function f : (X,d_x) → (Y,d_y) is continuous at a if and only if it is sequentially continuous at a.
5. Let (f_n) be a sequence of functions f_n : X → Y . If each fn is continuous at a ∈ X and fn → f uniformly, then show that f : X → Y is continuous at a .

GROUP C

ANSWER ANY TWO QUESTIONS FROM THE FOLLOWING(2×12=24)

1. a. let P:C→ C be a non constant polynomial then prove that there exist a r∈ C such that P(r)=0.

b. Consider the power series $a_0+a_1(z-c)+a_2(z-c)^2+$…………………… where $\lim\limits_{n\to\infty}\frac{|an|}{|an+1|}$ =R exists .then prove that R is the radius of convergence of the power series .

2. a. State and prove the Cauchy integral formula .

b. State and prove Cauchy Goursat formula.

3. Prove that A metric space (M, d) is connected if and only if the only subsets of M that are both open and closed are M and $\emptyset$. Equivalently, (M, d) is disconnected if and only if it has a non-empty, proper subset that is both open and closed.

4. a . Let (A, d_a) and (B, d_b) be metric spaces, and suppose that $f : A \to B$ be a continuous function from A to B. If A is connected, then show that its image f(A) is also connected.

b. Let (M, d) be a metric space and suppose that $\{X\alpha \mid \alpha \in A\}$ is a collection of subsets of M. If each X_α is connected and $\cap X\alpha \neq \emptyset$, then show that the union $\cup X\alpha$ is also connected.

Mathematics
MATHSEC – II/ C Programming Language
4th Semester / FM – 60

Group – A

[Answer any four questions given bellow **(4 X 3 = 12)**]

1. Why are the following unacceptable as C real constants:
   (i) 32342 (ii) 2,345.43 (iii)7.3E7.
2. Write the following values in standard exponential form:
   (i) 34215 (ii) 0.74 X 10^{-12} (iii) 43 X 10^{7}.
3. Write a C program to find the area of a triangle using the formula $\Delta = \frac{1}{2} ab \sin C$.
4. Write the C statements for the following:
   If a is less than or equal to b then store 10 in c; otherwise store 5 in c.
5. Write a do-while loop to evaluate and print the values of the cubic polynomial $y = x^3 - 3x + 1$ for the values of x =1.0 to 2.0, steps 0.2.
6. Two natural numbers $a, b\ (> a)$ are given. Write the C statements which returns the remainder when b is divided by a without using % operator.

Group – B

[Answer any four questions given bellow **(4 X 6 = 24)**]

7. Three positive numbers are given. Write a C program to determine whether these numbers form a triangle and if a triangle is formed, evaluate its area.
8. The p_H value of a solution is given by the formula $p_H = -\log_{10}[H^+]$, where $[H^+]$ is the hydrogen ion concentration of the solution. Write a C program to read $[H^+]$ and evaluate p_H.
9. Write a C program to evaluate whether a given natural number is prime.
10. N natural numbers are given. Write a C program which sorts these numbers in ascending orders.
11. Write a C program to solve a quadratic equation.
12. Write a C program to determine the product of two matrices.

Group – C

[Answer any two questions given bellow **(2 X 12 = 24)**]

13. (i). Write a C program to evaluate the partial sum of the exponential series $1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots$ at $x = 1.3$ up to 10,000 terms.

    (ii). Write a C program to determine all prime numbers between two given positive integers. (6+6)

14. (i). N is a given natural number. Write a C program to evaluate its factorial value.

    (ii). Write a C program to print all the Fibonacci numbers less than 1000. (6+6)

15. (i). Write a C program to approximate π correct up to 15 decimal places using Gregory's series.

    (ii). A square matrix is given. Write a C program to evaluate its trace. (6+6)

16. (i). Write a C program to evaluate the partial sum of the series $1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \cdots$ at $x = 0.57$ correct up to 8 decimal places.

    (ii). Write a C program to find the GCD and LCM of two given natural numbers. (6+6)

MATHEMATICS

MATH CC-XI (GROUP THEORY II)

5^{TH} SEMESTER /F.M -60

GROUP A

ANSWER ANY FOUR QUESTION FROM THE FOLLOWING(4×3=12)

1. Define automorphism with example .
2. What is inner automorphism? Show that Inn G is a normal subgroup of Aut G.
3. Prove that every characteristic subgroup are normal subgroup.
4. Prove that $Z_m \times Z_n \cong Z_{m\,n}$ iff (m,n)=1
5. What is the class equation of the group S_3.
6. Find all finite groups which have exactly two conjugacy classes.

GROUP B.

ANSWER ANY FOUR QUESTION FROM THE FOLLOWING(4×6=24)

1. Find AutG when G is an infinite cyclic group.
2. Prove that every group of order p^2 is abelian (p is prime).
3. Show that no group of order 105 is a simple group.
4. Let g be a finite group and a$\in G$ then prove that [G:C(a)]=$|cl(a)|$
5. Show that AutG (set of all automorphism of G) is a group.
6. State and prove first sylow's theorem.

GROUP C

ANSWER ANY TWO QUESTION FROM THE FOLLOWING(2×1=4)

1.a. Let G be a finite group and p be a prime integer .if p divides $|G|$ then prove that G has an element of order p and hence a subgroup of order p.

b. Find all groups of order 22.

2. a.What is commutator subgroup? "Commutator subgroup is normal subgroup"- justify this statement

b. Prove that every group is isomorphic to a subgroup of some semmetric group.

3.a. Let G be a group of order p^n , whwere p is prime ,n> 0.Prove that any subgroup of order p^{n-1} is a normal subgroup of G.

b. Show that A_4 is not a simple group

4.a. State and prove Fundamental theorem of finite abelian group.

b. Express The group of units modulo n as external direct products.

Mathematics
MATHCC – XII/ Numerical Methods
5^{th} Semester / FM – 40

Group – A

[Answer any five questions given bellow **(5 X 1 = 5)**]

1. Determine the number of significant figures of a number whose approximate value is 0.0562 and absolute error is 0.2×10^{-3}.
2. Prove that $\Delta \log f(x) = \log\left\{1 + \frac{\Delta f(x)}{f(x)}\right\}$.
3. ∇ and E are backward and shift operator respectively. Show that $E = \frac{1}{1-\nabla}$.
4. A function f, defined on [0,1], is such that $f(0) = 0, f\left(\frac{1}{2}\right) = -1, f(1) = 0$. Calculate the interpolating polynomial for $f(x)$.
5. Find the first term of the series whose second and subsequent terms are 15, 10, 7, 6, 7, 10.
6. Why polynomials are used for approximation in interpolation?
7. Give the geometrical interpretation of Newton-Raphson method.
8. Find the number of subintervals so that the maximum error in calculating $\int_{1.5}^{2.5} e^{-x}\,dx$ by Simpson's 1/3 rd rule is 0.5×10^{-6}.

Group – B

[Answer any three questions given bellow **(3 X 5 = 15)**]

9. Find the missing terms in the following table:

x	2.0	2.1	2.2	2.3	2.4	2.5	2.6
y	0.135	?	0.111	0.100	?	0.080	0.074

.

10. Construct Lagrange's interpolation polynomial by using the following data:

x	40	45	50	55
$y = f(x)$	15.22	13.99	12.62	11.13

Hence find $f(53)$.

11. Evaluate $\int_0^1 \frac{dx}{1+x}$ by Simpson's $\frac{1}{3}$ rd rule with step length = 0.25. Also calculate the absolute error.

12. Use Gauss elimination method to solve the following system of equations

$$-10x + 6y + 3z + 100 = 0$$
$$6x - 5y + 5z + 100 = 0$$
$$3x + 6y - 10z + 100 = 0$$

13. Find the value of $y(0.4)$using Runge-Kutta method of order 4 with $h = 0.2$, given that $\frac{dy}{dx} = \sqrt{x^2 + y}, y(0) = 0.8$.

Group – C

[Answer any two questions given bellow **(2 X 10 = 20)**]

1. (i). Find the relative error for evaluation of $u = x_1 x_2$ with $x_1 = 4.51, x_2 = 8.32$ having absolute error 0.01 in x_1 and 0.01 in x_2.

   (ii). Show that $\Delta^m \left(\frac{1}{x}\right) = \frac{(-1)^m m! h^m}{x(x+h)(x+2h)\ldots(x+mh)}$.

   (ii). Find the cube root of 10 upto 5 significant figures by Newton-Raphson method.

   (3+4+3)

2. (i). Define the k^{th} order difference of a function $f(x)$ and show that

$$\Delta^k f(x) = \sum_{i=0}^{k} (-1)^i \binom{k}{i} f(x + (k - i)h)$$

   where h is the step length.

   (ii). Find a positive real root of the equation $\cos x = 2x - 3$ correct upto three decimal places using fixed point iteration method. (5+5)

3. (i). Use power method to find the dominant eigenvalue and corresponding eigenvector of the matrix

$$\begin{pmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{pmatrix}.$$

   (ii). Explain Gauss-Seidel iterative method for solving a system of linear equations. State sufficient conditions for the convergence of the process. (5+(4+1))

4. (i). Deduce the iterative procedure $x_{n+1} = \frac{1}{2}\left(x_n + \frac{a}{x_n}\right)$ for evaluating $\sqrt{a}$ using Newton-Raphson method.

   (ii). Establish general quadrature formula from Newton's forward difference interpolating polynomial. Hence deduce Trapezoidal rule.. (4+(4+2))

Mathematics

MATHDSE-I/PROBABILITY AND STATISTICS
5th Semester/FM-60

GROUP- A

Answer any four questions from the following (4X3=12)

1. Let a random variable of continuous type have a pdf $f(x)$ whose graph is symmetric about the line $x = c$. If the mean value od X exist, then show that $E(X) = c$.

2. Let X and Y be discrete random variables. Then prove the following
(i) $E[E(y \mid X] = E(Y)$
(ii) $var(Y) = E[var(Y \mid X)] + var(E(Y \mid X))$.

3. Given the joint density function
$f(x, y) = \begin{cases} 2, & \text{if } 0 < y < x < 1 \\ 0, & \text{otherwise.} \end{cases}$ Find the regression equation of Y on X.

4. Find $P(0 < X < \frac{1}{3}, 0 < Y\frac{1}{3})$, if the random variables X and Y have joint probability density function
$$f(x, y) = \begin{cases} 4x(1-y), & \text{if } 0 < y < 1, 0 < x < 1 \\ 0, & \text{otherwise.} \end{cases}$$

5. If X be a continuous random variable with uniform distribution having mean 1 and $var(X) = 1$, find $P(X \leq 0)$.

6. State and prove Tchebycheff's inequality.

GROUP- B

Answer any four questions from the following (4X6=24)

7. (i) Show that the sample mean based on a simple random sample with replacement is an unbiased estimator of the population mean.
(ii) A coin is tossed 100 times. Find the probability of getting exactly 6 heads.

8. If X and Y be independent binomial variables with parameters (m, p) and (n, p) respectively, show that their sum $X+Y$ has a binomial distribution with parameters $(m+n, p)$.

9. A fair coin is tossed 4 times. Determine and sketch the cumulative distribution function for the following random variables
(i) $X(u)$, the number of "Heads" observed. (ii) $Y(u)$, the number of "Tails" observed. (ii) $D(u) = X(u) - Y(u)$, the difference between the number of heads and number of tails.

10. Let X be a poison random variable with parameter λ. Find the probability mass function of $Y = X^2 - 5$.

11. (i) If the two regression lines are $x+6y = 6$ and $3x+5y = 10$, find the correlation coefficient. (ii) Let the random variables X and Y have the following joint p. m. f. s (a) $P(X = x, Y = y) = 1/3$, if $(x, y) \in \{(0, 0), (1, 1), (2, 2)\}$ and 0 otherwise.

12. Find the moment generating function of normal distribution. Also find its mean and variance using moment generating function.

GROUP- C

Answer any two questions from the following (2X12=24)

13. The joint p. m. f. of X and Y is given by

$$P(X,Y) = \begin{cases} \frac{x+2y}{18}, & \text{if (x,y)} \in \{(1,1),(1,2),(2,1),(2,2)\} \\ 0, & \text{otherwise.} \end{cases}$$

(a) Find the marginal distributions.
(b) Verify whether X and Y are independent random variables.
(c) Find $P(X < Y)$, $P(X + Y > 2)$.
(d) Find the conditional p. m. f. of Y given $X = x, x = 1, 2$.

14. Let X and Y be two random variable with joint pdf

$$f(x,y) = \begin{cases} 5xy, & \text{if } 0 < x, y < 1 \\ 0, & \text{otherwise.} \end{cases}$$

(i) Find the moment generating function (mgf) of X and Y

(ii) Using mgf, compute $E(XY)$ and $E(X)$

(iii) compute $E(2X - 4XY)$.

15. Let $P(X)$ be the pmf of a random variable X. Find and sketch the cdf of X, where

$$P(X) = \begin{cases} \frac{x}{15}, & \text{if x} = 1,2,3,4,5 \\ 0, & \text{otherwise.} \end{cases}.$$

Also find

(i) $P(X = 1 or \ \ 2)$

(ii) $P(\frac{1}{2} < X < \frac{5}{2})$

(iii) $P(1 \leq X \leq 4)$.

16. (i) Prove that mean and variance of the Poisson distribution are same. Also find the mean and variance of standard exponential distribution.

(ii) Show that the Geometric random variable has memoryless property.

Mathematics

MATHDSE-II/NUMBER THEORY
5th Semester/FM-60

GROUP- A

Answer any four questions from the following (4X3=12)

1. Let a and b be integers, not both zero. Then a and b are relatively prime if and only if there exist integers x and y such that $1 = ax + by$.

2. For positive integers a and b $gcd(a, b)lcm(a, b) = ab$

3. Prove that the Diophantine Equation $x^4 + y^4 = z^2$ has no solution in integers.

4. Find all the positive integral solutions of the following Diophantine equation: $142x + 20y = 1000$.

5. Determine all solutions in the integers of the following Diophantine equations
(a) $56x + 72y = 40$
(b) $24x + 138y = 18$.

6. (i) State Fundamental Theorem of Arithmetic.
(ii) Solve the linear congruence $17x = 9(mod276)$.

GROUP- B

Answer any four questions from the following (4X6=24)

7. Let a, b be integers, not both zero. For a positive integer d, $d = gcd(a, b)$ if and only if (i) $d \mid a$ and $d \mid b$.
(ii) Whenever $c \mid a$ and $c \mid b$, then $c \mid d$.

8. The linear Diophantine equation $ax + by = c$ has a solution if and only if $d \mid c$, where $d = gcd(a, b)$. If X_0, Y_0 is are particular solution of this equation, then all other solutions are given by
$x = X_0 + \frac{b}{d}t$, $y = Y_0 - \frac{a}{d}t$, where t is an arbitrary integer.

9. (i) If $ca = cb$ (mod n), $then a$= b$(mod n/d)$, where $d = gcd(c, n)$.
(ii) If $ab(mod n)$, prove that $gcd(a, n) = gcd(b, n)$.

10. State and prove Chinese remainder theorem.

11. Let p be a prime and suppose that $p \nmid a$. Then prove that $a^{p-1} \equiv 1(mod p)$.

12. (i) Use Fermat's theorem to verify that 17 divides $11104 + 1$.
(ii) For $n > 2$, prove that $\phi(n)$ is an even integer.

GROUP- C

Answer any two questions from the following (2X12=24)

13. (i) Prove that an odd prime p is expressible as a sum of two square if and only if $p \equiv 1(mod 4)$.
(ii) Evaluate the Legendre symbol (3658/12703).

14. Find the solutions of each of the following systems of congruences
(i) $5x + 3y = 1(mod 7)$
$3x + 2y = 4(mod 7)$.
(ii) $7x + 3y = 6(mod 11)$
$4x + 2y = 9(mod 11)$.
(iii) $11x + 5y = 7(mod 20)$
$6x + 3y = 8(mod 20)$.

15. (i) Use Euler's theorem to establish the following
(a) For any integer a, $a^37 \equiv a(mod 1729)$.

(b) For any integer a, $a^13 \equiv a(mod 2730)$.

(ii) Prove that no prime p of the form $4k + 3$ is a sum of two squares.

16. (i) The quadratic congruence $x^2 + 1 = 0(mod p)$, where p is an odd prime, has a solution if and only if $p = 1(mod 4)$.
(ii) Show that $538 \equiv 4(mod 11)$.

MATHEMATICS

MATHCC –XIII (RING THEORY AND LINEAR ALGEBRA II)

SEMESTER 6^{TH}/F.M -60

GROUP A

ANSWER ANY FOUR QUESTIONS FROM THE FOLLOWIG (4×3=12)

1. Find the unit elements of Z[$\sqrt{-5}$].
2. Prove that in a PID R , a nonzero non unit element p is irreducible if and only if p is prime .
3. Show that Z[x] is not a PID.
4. Let F = R OR C and V be an inner product over F. For v, w ∈ R c∈ F. Show that$\|v - w\| \leq \|v\|$+$\|w\|$
5. Show that an orthogonal set of non zero vectors is linearly independent .
6. State and prove the Bessel's inequality .

GROUP –B

ANSWER ANY FOUR QUESTIONS FROM THE FOLLOWIG (4×6=24)

1. Let p be a non zero non unit element in an integral domain R the show that p is prime iff (p) is a non zero prime ideal of R.
2. Find a prime element in Z_{10} which is not irreducible.
3. Show that every Euclidean domain is PID.
4. If K is a field the show that K[x] is a Euclidean domain.
5. Let T be a linear operator on a finite dimensional vector space V. if f is the characteristic polynomial for T then show that f(T)=0.
6. Let T be a linear operator on V. if every subspace of V is invariant under T ,then T is a scalar multiple of the identity operator.

GROUP-C

ANSWER ANY TWO QUESTIONS FROM THE FOLLOWIG (2×12=24)

1. a. Let F be a field and f(x)∈ $F[x]$ with deg f(x)=2 or 3 .Then prove that f(x) is irreducible if and only if f(x) has no root in F.

b. Test the irreducibility of polynomials x^6+x^3+1 and $10x^3-7x+14$ over Q.

2. a. Let R be a commutative ring with identity such that R[x] is PID then prove that R is a field.

b. Determine all the units of Z[i].

3. a. Prove that on a finite dimensional inner product space of positive dimension , every self -adjoint operator has a nonzero characteristic vector .

b. Give an example of a two by two matrix A such that A^2 is normal but A is not normal.

4. Show that every finite dimensional vector space has an orthonormal basis and give an example .

MATHEMATICS

MATHCC-XIV (PARTIAL DIFFERENTIAL EQUATIONS AND APPLICATIONS)

6TH SEMESTER/FM-60

GROUP A

Answer any four questions from the following (4×3=12)

1. For each of the following, state whether the partial differential equation is linear, quasi-linear or nonlinear. If it is linear, state whether it is homogeneous or nonhomogeneous, and gives its order. (a) $u_{xx} + xu_y = y$, (b) $uu_x - 2xyu_y = 0$,
2. Find the general solution of $u_{xx} + u_x = 0$, by setting $u_x = v$.
3. Find the general solution of the linear equation $x^2 u_x + y^2 u_y = (x + y) u$.
4. Reduce each of the following equations $u_x - u_y = u$, $yu_x + u_y = x$, to canonical form, and obtain the general solution.
5. Find the solution of the following Cauchy problems: (a) $3u_x + 2u_y = 0$, with $u (x, 0) = \sin x$.
6. Determine the region in which the given equation is hyperbolic, parabolic, or elliptic, and transform the equation in the respective region to canonical form. (a) $xu_{xx} + u_{yy} = x^2$,

Group B

Answer any four questions from the following (4×6=24)

1. Show that $u (x, y; k) = e^{-ky} \sin (kx)$, $x \in R$, $y> 0$, is a solution of the equation $\nabla^2 u \equiv u_{xx} + u_{yy} = 0$.
2. Show that The general solution of a first-order, quasi-linear partial differential equation $a (x, y, u) u_x + b (x, y, u) u_y = c (x, y, u)$ is $f(\phi, \psi)=0$, where f is an arbitrary function of $\phi (x, y, u)$ and $\psi (x, y, u)$, and ϕ = constant = c1 and ψ = constant = c2 are solution curves of the characteristic equations $dx /a = dy /b = du/ c$.
3. State and prove The Cauchy Problem for a First-Order Partial Differential Equation.
4. . Solve the initial-value problem $u_x + 2u_y = 0$, $u (0, y)=4 e^{-2y}$. Using the method of separation of variable .
5. Find the characteristics and characteristic coordinates, and reduce the following equation to canonical form:) $u_{xx} + 2u_{xy} + 3u_{yy} + 4u_x + 5u_y + u = e^x$,
6. Given the parabolic equation $u_{xx} = au_t + bu_x + cu + f$, where the coefficients are constants, by the substitution $u = ve^{1/2bx}$, for the case $c = -b^2/4$, show that the given equation is reduced to the heat equation $v_{xx}= av_t+ g$, $g = fe^{-bx/2}$

GROUP C

Answer any two questions from the following (2×12=24)

1. Derive a. Heat Equation and

   b. Wave equation

2. a. Solve $u_{tt} =c^2u_{xx}$,

$u (x, t) = f (x)$ on $t = t(x)$,

$u (x, t) = g (x)$ on $x + ct = 0$, where $f (0) = g (0)$.

b. Obtain the solution of the initial-value problem of the homogeneous wave equation

$u_{tt} - c^2 u_{xx} = \sin(kx - \omega t)$, $-\infty < x < \infty$, $t > 0$,

$u(x, 0) = 0 = u_t(x, 0)$, for all $x \in R$, where c, k and ω are constants.

3 . a. Derive kepler's second law of motion.

b. Obtain the solution of the Cauchy problem $u_{xx} + u_{yy} = 0$,

$u(x, 0) = f(x)$ and

$u_y(x, 0) = g(x)$.

4 . a. Find the solution of the equation $u(x + y) u_x + u(x - y) u_y = x^2 + y^2$ with the Cauchy data $u = 0$ on $y = 2x$

b. Find the partial differential equation arising from each of the following surfaces: (i) $z = x + y + f(xy)$, (ii) $z = f(x - y)$, (iii) $z = xy + f(x^2+y^2)$.

Mathematics

MATHDSE-III/Point set topology
6th Semester/FM-60

GROUP- A

Answer any four questions from the following (4X3=12)

1. Let $f : A \to B$ be an injective function. Prove that A is countable if B is countable.

2. Let f be a map from a set A to its power set $P(A)$. Then prove that f is not surjective and $Card(A) < Card(P(A))$.

3. let $X = \{a, b, c, d\}$ be a topological space with the topology $\mathcal{T} = \{\phi, \{a\}, \{b\}, \{a, b\}, \{a, b, c, d\}\}$ and $A = \{b, c\}$ be a subset of X. Find limit points and interior points of A.

4. Define homeomorphism. Give example of two topological spaces which are not homeomorphic with justification.

5. Prove that a topological space X is connected iff $f : X \to \{0, 1\}$ is constant, where $\{0, 1\}$ has the discrete topology.

6. Show that sequences are continuous functions.

GROUP- B

Answer any four questions from the following (4X6=24)

7. (a) State and prove Hausdorff's maximal principle.
(b) Prove or give a counter example
(i) The union of infinitely many compact set is compact.
(i) A non-empty subset S of real numbers which has both a smallest and a largest element is compact.

8. (a) State and prove Baire category theorem.

(b) Define the terms: basis and subbasis.

(c) Let $X = \{a, b, c, d\}$ and $S = \{\{a, b, c\}, \{b, d\}\}$. Construct a topology on X using S.

9. Let $\{X_\alpha\}$ be an indexed family of spaces and $A_\alpha \subset X_\alpha$ for each α. If $\prod X_\alpha$ is given either the product or box topology, then show that $\prod \bar{A}_\alpha = \overline{\prod A_\alpha}$.

10. Define co-finite topology with example. Let X be a co-finite topological space and A be a subset of X. Find the closure of A.

11. Define Housedorff topological space. Prove that every metric space is a topological space. what about the converse? Justify your answer.

12. Show that continuous image of compact set is compact.

GROUP- C

Answer any two questions from the following (2X12=24)

13. (a) Define open and closed mapping.

(b) Give an example of open mapping which is not a closed mapping and closed mapping which is not open mapping.

(c) Let $X = \{a, b, c\}$ and $\mathcal{T} = \{\phi, X, \{a\}, \{b\}, \{c\}, \{a, b\}, \{b, c\}, \{c, a\}\}$ be a topology on X. Is $(X, \mathcal{T})$ a Hausdorff topological space? Justify your answer.

14. (a) Define open covering and compactness of topological space. Show that infinite discrete topological space is not connected.

(b) Let X and Y be two topological space and A is a connected subset of X. Let $f : X \to Y$ be a continuous function. Show that $f(A)$ is connected.

15. Let X and Y be two topological space and $f : X \to Y$. Show that the following are equivalent

(i) f is continuous.

(ii) For every subset A of X, one has $f(\bar{A}) \subset \overline{f(A)}$.

(iii) For any closed set B of Y, the set $f^{-1}(B)$ is closed in X.

16. (a) Prove that if a function f is a continuous on $[a, b]$ and $f(a) \neq f(b)$, then it assume every value between $f(a)$ and $f(b)$. 4

(b) Show that the function defined on $[0, 1]$ as $f(x) = 2x + 1, \forall x \in (0, 1]$ and $f(x) = 0$, if $x = 0$ does not satisfy the conclusion of intermediate Value theorem.

(c) Show that Q is not a connected topological space.

Mathematics
MATHDSE – IV/ Theory of Equations
6th Semester / FM – 60

Group – A

[Answer any four questions given bellow **(4 X 3 = 12)**]

1. Show that $(x^2 + x + 1)$ is a factor of $(x^{14} + x^7 + 1)$.
2. Apply Descartes' rule of signs to show that the equation $x^4 + 2x^2 - 7x - 5 = 0$ has two real roots and two imaginary roots.
3. Show that the equation $\frac{x^4}{4!} + \frac{x^3}{3!} + \frac{x^2}{2!} + \frac{x}{1!} + 1 = 0$ cannot have a multiple root.
4. If one of the roots of the equation $x^3 + px^2 + qx + r = 0$ equals the sum of the other two, then prove that $p^3 + 8r = 4pq$.
5. Show that the special roots of the equation $x^6 - 1 = 0$ are also roots of the equation $x^5 - x^4 + x^3 - x^2 + x - 1 = 0$.

Group – B

[Answer any four questions given bellow **(4 X 6 = 24)**]

6. (a). Show that the equation $x^3 - 2x - 5 = 0$ has no negative real root.
   (b). Show that the equation $x^n - nqx + (n-1)r = 0$ will have a pair of equal roots, if $q^n = r^{n-1}$. (3+3=6)
7. (a). Find the area of the triangle of which the lengths of the sides are the roots of the equation $x^3 - ax^2 + bx - c = 0$.
   (b). If α, β, γ be the roots of the equation $x^3 + 2x^2 + 1 = 0$, then find the equation whose roots are $\alpha + \frac{1}{\alpha}, \beta + \frac{1}{\beta}, \gamma + \frac{1}{\gamma}$. (3+3=6)
8. (a). If α be a root of the cubic $x^3 - 3x + 1 = 0$, then show that the other two roots are $(\alpha^2 - 2)$ and $(2 - \alpha - \alpha^2)$.
   (b). Show that $(x+1)^4 + a(x^4 + 1) = 0$ is a reciprocal equation, if $a \neq -1$. Solve it when $a = 2$. (3+3=6)
9. (a). Find the condition that the equation $x^4 + px^3 + qx^2 + rx + s = 0$ should have its roots $\alpha, \beta, \gamma, \delta$ connected by the relation $\alpha + \beta = 0$.
   (b). Solve the equation $x^3 - 9x + 28 = 0$ by Cardan's method. (3+3=6)
10. Find the equation whose roots are the six ratios of the roots of the cubic $x^3 + qx + r = 0$, $r \neq 0$. 6

11. Use Sturm's theorem to prove that the equation $x^3 - 7x + 7 = 0$ has two roots between 1 and 2 and one root between (-4) and (-3). 6

Group – C

[Answer any two questions given bellow **(2 X 12 = 24)**]

12. (a). Find the equation of the squared differences of the roots of the cubic $x^3 + x^2 - x = 1$. Hence show that two roots of this equation are equal.
    (b). Remove the second term of the equation $x^4 + 4x^3 - 7x^2 - 22x + 24 = 0$ and hence solve it. (6+6=12)
13. (a). Solve the equation $x^4 + 3x^3 + x^2 - 2 = 0$ by Ferrari's method.
    (b). If α, β, γ be the roots of the equations $4x^3 - 8x^2 - 19x + 38 = 0$, then find the equation whose roots are $\alpha - 2, \beta - 2, \gamma - 2$. Solve the obtained equation and from it find the roots of the original equation. (6+6=12)
14. (a). If $\alpha, \beta, \gamma, \delta$ be the roots of the equation $x^4 + px^3 + qx^2 + rx + s = 0$, then find, in terms of p, q, r, s, the value of $\sum \frac{\alpha\beta}{\gamma^2}$.
    (b). Show that the equation $x^4 + x^3 - 4x^2 - 3x + 3 = 0$ can be transformed into a reciprocal equation by increasing the roots by 2. Solve the reciprocal equations and hence obtain the solution of the given equation.
15. (a). Prove that the special roots of the equation $x^9 - 1 = 0$ are the roots of the equation $x^6 + x^3 + 1 = 0$ and their values are $\cos\frac{2r\pi}{9} \pm \sin\frac{2r\pi}{9}$, $r = 1, 2, 4$.
    (b). If α, β, γ be the roots of the cubic $x^3 - 21x + 35 = 0$, then show that $(\alpha^2 + 2\alpha - 14)$ is equal to either β or γ.